Mathematics process categories Form A

All of the UK curricula define multiple categories of mathematical proficiency that require students to be able to use and apply mathematics, beyond simple recall of facts and standard procedures. While the intentions are very similar, the terminology varies between regions. *Progress Test in Maths (PTM)* categories are based on the Aims in the KS1, KS2 and KS3 *National Curriculum for England*, and are also comparable with the GCSE Assessment Objectives, adopting some language from both. The main change has been to divide 'Fluency' into two strands.

FF: Fluency in facts and procedures

Students can, for example:

- recall mathematical facts, terminology and definitions (such as the properties of shapes);
- recall number bonds and multiplication tables;
- perform straightforward calculations.

FC: Fluency in conceptual understanding

Students can, for example:

- demonstrate understanding of a mathematical concept in the context of a routine problem (e.g. calculate with or compare decimal numbers, identify odd numbers, prime numbers, multiples);
- extract information from common representations, such as charts, graphs, tables and diagrams;
- identify and apply the appropriate mathematical procedure or operation in a straightforward word problem (for example, knowing when to add, multiply or divide).

MR: Mathematical reasoning

Students can, for example:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- interpret and communicate information accurately.

PS: Problem solving

Students can, for example:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;

- evaluate methods used and results obtained;
- evaluate solutions to identify how they may have been affected by assumptions made.

There is a limit to how thoroughly MR and PS can be assessed in a short, whole-curriculum test such as *PTM*. Teachers are urged to ensure that their curriculum includes a balanced diet of extended tasks, investigations, problem solving and collaborative activities.

These tables show how the questions in *PTM*14 map onto these process categories.

Paper test		
Process category	**Mental Maths**	**Applying and Understanding Maths**
FF: Fluency in facts and procedures	1, 2, 3, 9, 11, 12, 15	1
FC: Fluency in conceptual understanding	4, 5, 6, 7, 8, 10, 13, 14, 16, 17, 18, 19, 20	2, 3, 4, 5, 11, 13
MR: Mathematical reasoning		6, 7, 8, 9, 10, 12, 14, 15, 16, 18
PS: Problem solving		17, 19, 20

Digital test		
Process category	**Mental Maths**	**Applying and Understanding Maths**
FF: Fluency in facts and procedures	1, 2, 3, 9, 11, 12, 15	1a, 1b
FC: Fluency in conceptual understanding	4, 5, 6, 7, 8, 10, 13, 14, 16, 17, 18, 19, 20	2, 3, 4, 5, 14a, 14b, 14c, 16a, 16b, 18, 19
MR: Mathematical reasoning		6a, 6b, 6c, 7a, 7b, 8, 9, 10a, 10b, 11, 12a, 12b, 13, 15a, 15b, 17, 20, 22a, 22b
PS: Problem solving		21, 23a, 23b, 24a, 24b, 24c, 24d

Mathematics process categories in Wales, Scotland and Northern Ireland

The process categories are based on the National Curriculum and GCSE syllabuses for England. The curricula for Wales, Scotland and Northern Ireland have similar requirements, although there is wide variation in the way they are defined.

Wales	Closest *PTM* process categories			
Key Stage 3 Skills	**FF**	**FC**	**MR**	**PS**
1. Solve Mathematical Problems				•
2. Communicate Mathematically		•	•	
3. Reason Mathematically		•	•	
Key Stage 3 Range	•			

Northern Ireland	Closest *PTM* process categories			
Key Stage 3 Using Mathematics	**FF**	**FC**	**MR**	**PS**
Communicate		•	•	
Manage Information			•	
Think Critically		•	•	
Solve Problems and Make Decisions				•
Individual mathematical topics	•			

Scotland	Closest *PTM* process categories			
Experiences and outcomes	**FF**	**FC**	**MR**	**PS**
develop a secure understanding of the concepts, principles and processes of mathematics and apply these in different contexts, including the world of work			•	•
engage with more abstract mathematical concepts and develop important new kinds of thinking			•	
understand the application of mathematics, its impact on our society past and present, and its potential for the future				
develop essential numeracy skills which will allow me to participate fully in society	•			
establish firm foundations for further specialist learning	•	•		
understand that successful independent living requires financial awareness, effective money management, using schedules and other related skills			•	•
interpret numerical information appropriately and use it to draw conclusions, assess risk, and make reasoned evaluations and informed decisions				•
apply skills and understanding creatively and logically to solve problems, within a variety of contexts			•	•
appreciate how the imaginative and effective use of technologies can enhance the development of skills and concepts				

Education Scotland: “Curriculum for Excellence: Numeracy and Mathematics” 14 May 2009.

Assessment for learning: following up the test activities

Each *PTM* assessment test is designed to align with the mathematics curriculum at a level appropriate for the students in the relevant age group. The activities may therefore be used to obtain diagnostic information about each student's strengths and weaknesses, and may also be used to provide a basis from which students' mathematical understanding may be further developed.

This section discusses some of the ways in which students may be helped to improve areas of weakness and to build on what they already know in order to deepen their understanding. These notes cover only a few of the possibilities. In talking to students and discussing the activities on which they did well, as well as those they were unable to complete correctly, you may find approaches that are helpful to them, building on their own strengths and interests.

You will need to refer to the activities in the *Student Booklet* and the Teacher's script in the *At a Glance Guide* when reading these notes, as they form the basis of the ideas suggested. The activities are referred to here by both their numbers and their names.

Formative notes on the questions

The standardised total scores on *PTM* give you an indication of the *overall* performance of your students, and a basis for progress monitoring. This section is intended to you help identify the specific difficulties that students have with individual questions, and to suggest possible activities to help guide your future teaching.

Mental Maths test

These questions test students' basic number skills and recall of facts. If students score poorly, it may be that they simply lack these skills, and are relying too heavily on written methods for even simple calculations. They may lack the confidence to recall mathematical facts under pressure.

Regular quick-fire quizzes may help students gain fluency and confidence, and there are many software packages that allow students to practice skills in the context of games.

However, these should not displace problem-solving and investigative mathematics activities, which can also help students gain fluency by fostering a deeper understanding of mathematical concepts and their connections, reducing their dependence on 'memorising' fragments of information.

Paper Test

Question 1: Fractions

In this question students are asked to add two fractions and subtract a fraction from a mixed number, giving their answers in their simplest form.

The addition question (part a) has two fractions with different denominators and, as the total is greater than one, it needs to be expressed as a mixed number.

In the second question (part b) the fraction and the mixed number, again have different denominators.

Many students find working with fractions difficult, so regular practice can be helpful.

Using fractions and mixed numbers rather than decimals when studying other topics such as area and perimeter or averages can be useful.

Question 2: Transformation

This problem shows two congruent triangles: A and B, drawn on a coordinate grid. The task is to describe the transformation which maps shape A onto shape B.

Students need lots of practice using real shapes and tracing paper as they to learn to describe transformations and to draw transformed shapes.

Question 3: Calculations

Part a of this task requires students to interpret and compare the size of numbers written in the standard form $A \times 10^n$ where $1 \leq A \leq 10$ and n is a positive or negative integer. In part b, they are asked to divide one such number by another, and give their answer in standard form.

Students need to be able to work with numbers in standard form, both with and without a calculator. It is helpful if students understand that writing numbers in this form is useful by working with very large and very small numbers in real life situations.

Question 4: Brackets

In this question, students are given an expression in the form of the product of two brackets containing linear expressions. They are also given a list of a further six expressions, some of which are equivalent to the original expression. The task is to identify any equivalent expressions. The list contains three equivalent expressions and three expressions which contain common errors students make when expanding brackets.

Expanding products of binomials is a topic that students find quite difficult. Pairing up equivalent fractions and correcting incorrect 'homework' are approaches which can provide some variety when practising multiplying out brackets.

Question 5: Nets

In this task, three nets of cuboids are shown. These need to be paired up with three of four given cuboids. Students need to recognise, describe and build simple 3D shapes, including making nets.

In the classroom, students need to practice cutting through the edges of 3D shapes to make nets before building a variety of 3D shapes from nets.

If students find this task difficult, they might first pair up the cube with the net that contains only square shapes. Then the net with small squares top and bottom could be matched with the 3D shape top right because it is the only 3D shape with two small square faces and four long narrow rectangular sides.

Question 6: Puppies

In this problem, students are given the weight of a puppy when it was born and asked to find its weight at the end of the first month and then after the second month. This problem requires students to increase a value by a percentage (part a) and then by a fraction of the new amount (part b). They are also asked to show their working in part b.

Many students solve such problems by finding the percentage increase and adding it onto the original. As this is not a reversible process it is a good idea to go on to find the total new percentage in one step as this will help later when learning to find the original value given the value found after an increase or decrease.

When finding a fraction of a value it is useful to realise that the word 'of' implies multiplication. Most students understand that if you want 2 of a value you multiply by 2. It also helps if students realise that this is true for fractions.

Question 7: Perimeter Sequence

In this task, a table containing a sequence of four square shapes, increasing in size, with their shape number and perimeter. Students are asked to find the perimeter of the tenth shape (part a) and a formula for the perimeter of the *n*th term (part b). Looking for the pattern to find how the shape 'grows' without finding all the terms in between can help students to find the *n*th term. After looking at the four shapes shown, students may see that the shape 'grows' by adding three sides of a square;

this means that the perimeter increases in steps of 2cm because at each step 1cm is 'lost'.

When working on such tasks in the classroom, students need to be encouraged to describe how they see the shapes 'grow' – one student's vision may be very different to another student's vision. To find the correct answer to part a of this task, a student may decide that there are nine steps of 2 which need to be added to the starting point of 4, making 22. So the correct answer to part b is (n – 1) multiplied by 2 and then 4 more, making 2n + 2.

Question 8: Swimming Race

Three graphs drawn on the same coordinate grid show the distances three swimmers are from the starting point of a race as time passes.

The questions ask who won the race and how long she took (part a), and at what time one of the swimmers overtook another (part b).

Interpreting information when several graphs are drawn on the same axes can be quite difficult. A common error in this question is to identify the winner as the person who took the longest time.

Interpreting what graphs show and obtaining information from them are important skills which need practice in many different contexts. All too often students are asked to draw graphs, rather than discussing what information is shown in graphs that can be found in magazines and newspapers.

Question 9: Test Results

This task provides a scatter graph showing the scores for a group of students who all took two tests. Five statements relating to how the graph can be interpreted are shown, with some words missing. The spaces need to be filled in using words or numbers from a given list.

In the classroom, activities of this sort can help students realise how much information can be found when interpreting what a graph shows.

In order to answer this type of question very careful reading, and understanding the information shown in the graph are both needed. It may be that students find this task difficult because they are not often asked to complete tasks of this sort.

Question 10: Sequence

In this question the first four terms of a number sequence are provided and students are asked to find the 10th term (part a), an algebraic expression for the *n*th term (part b) and explain why a given number cannot be a member of the sequence (part c).

The numbers are the first four terms of an arithmetic sequence, the pattern is 'add 4'.

To find the 10th term, we add nine fours to six (42), and the *n*th term is 4(n-1) + 4 or 4n + 8. Part c asks students to explain why 401 cannot be a term in the sequence. A variety of answers is acceptable, including 'because it is an odd number'.

Question 11: Number Machine

Here we have a number machine made up of three functions, a subtraction, a squaring and another subtraction.

Students are asked to work from left to right to find the result, *y*, of putting a value, *x*, through the machine (part a). They then need to reverse the process to find the start value, *x*, given the outcome, *y* (part b). Finally they are asked to write the equation which this machine represents (part c).

Number machines are a useful way of leading into algebra.

Reversing the process requires the use of opposite functions, in this case subtraction with addition and squaring with square rooting. It is also necessary to use the functions in the reverse order. Seeing this in a number machine can help towards learning to solve algebraic equations.

Question 12: Flower Garden

In this problem, students are presented with a diagram of a circular flower bed surrounded by a circular lawn. The diameter and the width of the lawn are given together with the formula for the area of a circle and a value for pi.

Students are asked to find the area of the flower bed (part a) and the area of the lawn (part b).

Before finding the area of the flower bed using the formula provided, students need to determine its radius by finding half of 6m and then subtracting 1m. They are also asked to write their answer correct to one decimal place.

To find the area of the lawn, students need to calculate the area of the lawn when it includes the flower bed and then subtract the area of the flower bed. Students are asked to show their working in part b.

Question 13: Poster

In this question a picture of a yacht is shown together with its enlargement to make a poster. The height and width of the picture are shown, but only the width of the poster is given. The first task is to find the height of the poster (part a). Additional information is then provided, the height of the boat's mast on the poster is given and students are asked to calculate the height of the boat's mast on the picture (part b).

This question can be approached in various ways. Using the concepts of ratio or scale factor, comparing the widths of the two diagrams students may see that the poster is an enlargement of the picture by a scale factor of 2.5. This can then be used to find both the height of the poster, by multiplication, and the height of the mast, by division.

Question 14: Pace Length

In this problem, students are provided with a formula which shows the approximate relationship between the number of steps taken every minute and the pace length. The number of steps Tom takes per minute is given and his pace length is to be calculated. Students are asked to show their working.

This is not a simple formula to use because the required variable is in the denominator. Some students may rearrange the formula in order to make the required variable the subject of the formula ($p = n/144$), but many students find this very difficult. Alternatively, the given value can be substituted into the formula first ($72/p = 144$) before finding p. It may be, that students can find the value of p by trying one or two values, or asking themselves 'What must I divide p by to get 144?'.

Question 15: Vacuum Cleaner

In this task we are provided with a scale drawing of an empty room and the position of a power socket is shown. Jess uses a vacuum cleaner with a power lead of a given length.

Students are asked to make an accurate drawing of the area of the room which Jess can vacuum.

To answer this question, students need to be able to find the locus of an object that moves according to a rule. They need to realise that the perimeter of the area Jess can vacuum is the arc of a circle, where the centre is the socket and the radius is 5 metres. They then need to calculate the radius of the circle using the given scale of the drawing, and draw this accurately using compasses. It may be that some students do not get sufficient practice of drawing geometric diagrams and constructions accurately.

Question 16: Toothpaste

This problem shows a cylinder of toothpaste and provides the formula for the volume of a cylinder and the dimensions of the cylinder. The task is to find the volume of the toothpaste in the cylinder. The diameter of the cylinder is given so this needs to be halved to find the radius needed in the formula. Both of the dimensions are less than a whole centimetre so the question uses small, decimal values. The answer is to be given correct to two decimal places and the work needs to be shown.

Some students make avoidable errors when multiplying and dividing by decimals. These errors can often be self-corrected if students are encouraged to ask themselves whether their answers make sense.

In the classroom, this question could be opened out to find the volume of a tube of toothpaste and, find the length of time the tube could last.

Question 17: Roof

A diagram showing the cross section of a roof is given. The cross section is an isosceles triangle with its base length and vertical height given. The problem is to calculate the length of the slope of the roof.

In this problem students need to use Pythagoras' Theorem to calculate the hypotenuse.

Students need to see that half of the diagram forms a right-angled triangle and that half the base length should be used in the calculation.

This problem requires students to use Pythagoras' Theorem in its most straightforward form, but the numbers involved are decimals, so they need to take care when considering the positions of the decimal points as they square the sides and find the square root of the sum.

Question 18: School Trips

This task provides a tree diagram showing the different places that children will be going to on a school trip. At random, half of the group will be chosen to go to the countryside and half to the city; these probabilities are already entered on the diagram.

Further information is given about the places the children will visit and the question requires students to fill in the empty boxes on the tree (part a). They are then asked to calculate the probability that a child visits one particular place (part b).

At random, children are selected to visit three different places in the countryside and four different places in the city, so different fractions will need to be entered. Entering the probabilities in the boxes is comparatively easy: the fraction to be entered in the three top boxes is $\frac{1}{3}$, and $\frac{1}{4}$ in the bottom four boxes. The probability that Freya will go to the art gallery is more complex, as the fractions $\frac{1}{2}$ and $\frac{1}{4}$ need to be multiplied together to give $\frac{1}{8}$. Some students find calculating the probability of compound events difficult, and tend to add the two fractions rather than multiplying them.

Question 19: Sorting Functions

In this task, diagrams of three graphs – two line graphs and one quadratic graph – are shown. Four equations are given: two are linear and two are quadratic, and students are asked to write the correct equation under each graph (part a). A table of values for *x* and *y* is provided and students are asked which graph it represents (part b).

Students are often asked to draw graphs rather than matching graphs with their equations, so this is a different way of assessing their understanding of graphical concepts. In part b, understanding that the equation of a straight line graph with a positive gradient has a positive x coefficient when expressed in the form y = ax + b, helps to identify graphs A and therefore B. Understanding that two values of x (one positive and one negative) satisfy each value of y, helps to identify graph C, which is y= x2.

Part b, can be answered by substituting in one or more of the equations provided. Alternatively, by looking at the table, it can be seen that as *x* goes from negative to positive, *y* also goes from negative to positive, so the graph goes from the third to the first quadrant: also the *y* values are two less than the *x* values, so the table represents graph A.

In the classroom, trying to imagine what a graph is going to look like before drawing it, is an interesting activity which can lead to a much deeper understanding.

Question 20: Pentagon

In this question, a regular pentagon with three diagonals is drawn. Students are asked to calculate one of the interior angles (part a), to name two congruent triangles (part b) and to calculate the size of two of the angles formed by the diagonals (parts c and d).

The interior angle of a regular pentagon can be found using a number of different approaches. One method requires student to know that the exterior angle of a regular pentagon is 360°/5 (72°) and that the sum of the angles on a straight line is 180°, so each interior angle is 108°.

The diagonals already drawn show three congruent triangles, formed by two adjacent sides of the pentagon and a diagonal.

The sizes of the required angles can be calculated using the properties of the angles in an isosceles triangle.

Many geometry questions can be calculated using several methods. In the classroom, it is interesting to challenge different groups to use different methods, maybe giving them a rule which must be used for example: ‘the angles in a triangle add up to 180°’ or ‘the exterior angles of a polygon add up to 360°’.

Digital Test

Question 1: Fractions

In this question students are asked to add two fractions and subtract a fraction from a mixed number, giving their answers in their simplest form.

The addition question (part a) has two fractions with different denominators and, as the total is greater than one, it needs to be expressed as a mixed number.

In the second question (part b) the fraction and the mixed number, again have different denominators.

Many students find working with fractions difficult, so regular practice can be helpful.

Using fractions and mixed numbers rather than decimals when studying other topics such as area and perimeter or averages can be useful.

Questions 2 and 3: Calculations

Part a of this task requires students to interpret and compare the size of numbers written in the standard form A x 10n where 1 ≤ A ≤ 10 and n is a positive or negative integer. In question 3, they are asked to divide one such number by another, and give their answer in standard form.

Students need to be able to work with numbers in standard form, both with and without a calculator. It is helpful if students understand that writing numbers in this form is useful by working with very large and very small numbers in real life situations.

Question 4: Brackets

In this question, students are given an expression in the form of the product of two brackets containing linear expressions. They are also given a list of a further six expressions, some of which are equivalent to the original expression. The task is to identify any equivalent expressions. The list contains three equivalent expressions and three expressions which contain common errors students make when expanding brackets.

Expanding products of binomials is a topic that students find quite difficult. Pairing up equivalent fractions and correcting incorrect 'homework' are approaches which can provide some variety when practising multiplying out brackets.

Question 5: Nets

In this task, three nets of cuboids are shown. These need to be paired up with three of four given cuboids. Students need to recognise, describe and build simple 3D shapes, including making nets.

In the classroom, students need to practice cutting through the edges of 3D shapes to make nets before building a variety of 3D shapes from nets.

If students find this task difficult, they might first pair up the cube with the net that contains only square shapes. Then the net with small squares top and bottom could be matched with the 3D shape top right because it is the only 3D shape with two small square faces and four long narrow rectangular sides.

Question 6: Puppies

In this problem, students are given the weight of a puppy when it was born and asked to find its weight at the end of the first month and then after the second month. This problem requires students to increase a value by a percentage (part a) and then by a fraction of the new amount (part b). Students are then asked to give the weight of the puppy after the second month (part c).

Many students solve such problems by finding the percentage increase and adding it onto the original. As this is not a reversible process it is a good idea to go on to find the total new percentage in one step as this will help later when learning to find the original value given the value found after an increase or decrease.

When finding a fraction of a value it is useful to realise that the word 'of' implies multiplication. Most students understand that if you want 2 of a value you multiply by 2. It also helps if students realise that this is true for fractions.

Question 7: Perimeter Sequence

In this task, a table containing a sequence of four square shapes, increasing in size, with their shape number and perimeter. Students are asked to find the perimeter of the tenth shape (part a) and a formula for the perimeter of the nth term (part b).

Looking for the pattern to find how the shape 'grows' without finding all the terms in between can help students to find the nth term. After looking at the four shapes shown, students may see that the shape 'grows' by adding three sides of a square; this means that the perimeter increases in steps of 2cm because at each step 1cm is 'lost'.

When working on such tasks in the classroom, students need to be encouraged to describe how they see the shapes 'grow' – one student's vision may be very different to another student's vision. To find the correct answer to part a of this task, a student may decide that there are nine steps of 2 which need to be added to the starting point of 4, making 22. So the correct answer to part b is $(n - 1)$ multiplied by 2 and then 4 more, making $2n + 2$.

Questions 8 and 9: Swimming Race

Three graphs drawn on the same coordinate grid show the distances three swimmers are from the starting point of a race as time passes.

The questions ask who won the race and how long she took (question 8), and at what time one of the swimmers overtook another (question 9).

Interpreting information when several graphs are drawn on the same axes can be quite difficult. A common error in this question is to identify the winner as the person who took the longest time.

Interpreting what graphs show and obtaining information from them are important skills which need practice in many different contexts. All too often students are asked to draw graphs, rather than discussing what information is shown in graphs that can be found in magazines and newspapers.

Questions 10 and 11: Test Results

This task provides a scatter graph showing the scores for a group of students who all took two tests. Students are asked for the range of scores in Test A (question 10a) and to then find the student with the highest score on Test A and work out what their score would have been on Test B (question 10b).

In question 11, three statements relating to how the graph can be interpreted are shown, with some words missing. The spaces need to be filled in using words from a given list.

In the classroom, activities of this sort can help students realise how much information can be found when interpreting what a graph shows.

In order to answer this type of question very careful reading, and understanding the information shown in the graph are both needed. It may be that students find this task difficult because they are not often asked to complete tasks of this sort.

Questions 12 and 13: Sequence

In this question the first four terms of a number sequence are provided and students are asked to find the 10th term (question 12a), an algebraic expression for the nth term (question 12b) and choose which numbers from a given list cannot be a member of the sequence (question 13).

The numbers are the first four terms of an arithmetic sequence, the pattern is 'add 4'.

To find the 10th term, we add nine fours to six (42), and the nth term is $4(n-1) + 4$ or $4n + 8$.

Question 14: Number Machine

Here we have a number machine made up of three functions, a subtraction, a squaring and another subtraction.

Students are asked to work from left to right to find the result, y, of putting a value, x, through the machine (part a). They then need to reverse the process to find the start value, x, given the outcome, y (part b). Finally they are asked to choose the equation which this machine represents (part c).

Number machines are a useful way of leading into algebra.

Reversing the process requires the use of opposite functions, in this case subtraction with addition and squaring with square rooting. It is also necessary to use the functions in the reverse order. Seeing this in a number machine can help towards learning to solve algebraic equations.

Question 15: Flower Garden

In this problem, students are presented with a diagram of a circular flower bed surrounded by a circular lawn. The diameter and the width of the lawn are given together with the formula for the area of a circle and a value for pi.

Students are asked to find the area of the flower bed (part a) and the area of the lawn (part b).

Before finding the area of the flower bed using the formula provided, students need to determine its radius by finding half of 6m and then subtracting 1m. They are also asked to write their answer correct to one decimal place.

To find the area of the lawn, students need to calculate the area of the lawn when it includes the flower bed and then subtract the area of the flower bed. Students are asked to show their working in part b.

Question 16: Poster

In this question a picture of a yacht is shown together with its enlargement to make a poster. The height and width of the picture are shown, but only the width of the poster is given. The first task is to find the height of the poster (part a).

Additional information is then provided, the height of the boat's mast on the poster is given and students are asked to calculate the height of the boat's mast on the picture (part b).

This question can be approached in various ways. Using the concepts of ratio or scale factor, comparing the widths of the two diagrams students may see that the poster is an enlargement of the picture by a scale factor of 2.5. This can then be used to find both the height of the poster, by multiplication, and the height of the mast, by division.

Question 17: Pace Length

In this problem, students are provided with a formula which shows the approximate relationship between the number of steps taken every minute and the pace length.

Students are given a table showing number of paces per minute and pace length and they are asked to use the formula to fill in the missing numbers in the table.

This is not a simple formula to use because the required variable is in the denominator. Some students may rearrange the formula in order to make the required variable the subject of the formula (p = n/144), but many students find this very difficult. Alternatively, the given value can be substituted into the formula first (72/p = 144) before finding p. It may be, that students can find the value of p by trying one or two values, or asking themselves 'What must I divide p by to get 144?'.

Questions 18 and 19

This question shows a flag shape, drawn on a coordinate grid. Students can move or change the shape by dragging the three vertices.

The first task is to translate the shape by 3 units to the right and 2 units down. The flag should be moved to its new position, but keep its size and shape. The second task is to enlarge the shape by a scale factor of 3 with the centre of enlargement at (0,1). The flag should become 6 units high by 3 units wide, without distorting its shape – but it should also be in the correct place so that each vertex lies on a straight line through the original vertex and the centre of enlargement at (0,1). Students need lots of practice using real shapes and tracing paper as they to learn to describe translations and rotations and to draw transformed shapes. Enlargements can be constructed on graph paper by projecting lines from the centre of enlargement through the vertices.

Question 20: Toothpaste

This problem shows a cylinder of toothpaste and provides the formula for the volume of a cylinder and the dimensions of the cylinder. The task is to find the volume of the toothpaste in the cylinder. The diameter of the cylinder is given so this needs to be halved to find the radius needed in the formula. Both of the dimensions are less than a whole centimetre so the question uses small, decimal values. The answer is to be given correct to two decimal places.

Some students make avoidable errors when multiplying and dividing by decimals.

These errors can often be self-corrected if students are encouraged to ask themselves whether their answers make sense.

In the classroom, this question could be opened out to find the volume of a tube of toothpaste and, find the length of time the tube could last.

Question 21: Roof

A diagram showing the cross section of a roof is given. The cross section is an isosceles triangle with its base length and vertical height given. The problem is to calculate the length of the slope of the roof.

In this problem students need to use Pythagoras' Theorem to calculate the hypotenuse.

Students need to see that half of the diagram forms a right-angled triangle and that half the base length should be used in the calculation.

This problem requires students to use Pythagoras' Theorem in its most straightforward form, but the numbers involved are decimals, so they need to take care when considering the positions of the decimal points as they square the sides and find the square root of the sum.

Question 22: School Trips

This task provides a tree diagram showing the different places that children will be going to on a school trip. At random, half of the group will be chosen to go to the countryside and half to the city; these probabilities are already entered on the diagram.

Further information is given about the places the children will visit and the question requires students to fill in the empty boxes on the tree (part a). They are then asked to calculate the probability that a child visits one particular place (part b).

At random, children are selected to visit three different places in the countryside and four different places in the city, so different fractions will need to be entered.

Entering the probabilities in the boxes is comparatively easy: the fraction to be entered in the three top boxes is 1/3 and 1/4 in the bottom four boxes. The probability that Freya will go to the art gallery is more complex, as the fractions 1/2 and 1/4 need to be multiplied together to give 1/8. Some students find calculating the probability of compound events difficult, and tend to add the two fractions rather than multiplying them.

Question 23: Sorting Functions

In this task, diagrams of three graphs – two line graphs and one quadratic graph – are shown. Four equations are given: two are linear and two are quadratic, and students are asked to put the correct equation under each graph (part a). A table of values for x and y is provided and students are asked which graph it represents (part b).

Students are often asked to draw graphs rather than matching graphs with their equations, so this is a different way of assessing their understanding of graphical concepts. In part b, understanding that the equation of a straight line graph with a positive gradient has a positive x coefficient when expressed in the form y = ax + b, helps to identify graphs A and therefore B. Understanding that two values of x (one positive and one negative) satisfy each value of y, helps to identify graph C, which is y= x2.

Part b can be answered by substituting in one or more of the equations provided. Alternatively, by looking at the table, it can be seen that as x goes from negative to positive, y also goes from negative to positive, so the graph goes from the third to the first quadrant: also the y values are two less than the x values, so the table represents graph A.

In the classroom, trying to imagine what a graph is going to look like before drawing it, is an interesting activity which can lead to a much deeper understanding.

Question 24: Pentagon

In this question, a regular pentagon with three diagonals is drawn. Students are asked to calculate one of the interior angles (part a), to name two congruent triangles (part b) and to calculate the size of two of the angles formed by the diagonals (parts c and d).

The interior angle of a regular pentagon can be found using a number of different approaches. One method requires students to know that the exterior angle of a regular pentagon is 360°/5 (72°) and that the sum of the angles on a straight line is 180°, so each interior angle is 108°.

The diagonals already drawn show three congruent triangles, formed by two adjacent sides of the pentagon and a diagonal.

The sizes of the required angles can be calculated using the properties of the angles in an isosceles triangle.

Many geometry questions can be calculated using several methods. In the classroom, it is interesting to challenge different groups to use different methods, maybe giving them a rule which must be used for example: 'the angles in a triangle add up to 180°' or 'the exterior angles of a polygon add up to 360°'.

Feedback to parents and carers

A report on the individual student is available to support feedback to parents or carers. This *Individual report for parents* strips away much of the technical detail that is included in the *Group report for teachers*. A series of statements, tailored for parents, is included to explain what the results mean and how learning may be affected. Recommendations focus on how the parent or carer can work with the school to support the student at home.

In addition to the *Individual report for parents*, you may wish to provide supporting information, either orally or in writing, explaining the process and outcomes. The following list provides you with guidelines to assist with this communication.

- Stress the school's commitment to identifying and addressing the needs of each individual student in order to understand and maximise their potential.
- Explain that testing with *PTM*14 is part of the school's regular assessment regime and that all students in the year group(s) have been tested.
- As part of the test, students were tested on their mental maths ability as well as their ability to apply and understand mathematics in a written context.
- You may wish to summarise the specific outcomes and recommendations from the test for that individual student (which are also shown on the *Individual report for parents*).
- Parents or carers should be reassured that if they have any questions or concerns or would like any further advice on how best to support their child, then they should contact the school.

A sample letter (Figure 1) is provided to support your communications with parents/carers after testing with *PTM*14.

Figure 1: Sample parent/carer feedback letter

Dear Parent or Carer,

In school, we wish to assess all our students to see what their needs are and how we can best help them learn and achieve.

As part of this process, your child has completed the Progress Test in Maths 14, which assesses key aspects of maths, such as shape, number and mathematical concepts (like money, place value and time).

A copy of the Individual report for parents is included*. This shows your child's results and describes what these mean in terms of the ways in which he/she will learn best and how you can support him/her at home.

[If the report is not included a relevant short extract can be included instead.]

If you have any queries or concerns please contact us.

Yours faithfully,

[School/Establishment name]

* If possible, it is helpful to parents to discuss the report with them on a suitable occasion before sending it out.